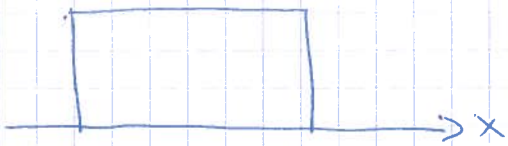
chap 8

exo 1

We consider

$$\frac{\partial h}{\partial t} + K h^2 \frac{\partial h}{\partial x} = 0 \quad \text{with } K = \frac{\rho g \sin\theta}{\mu}$$



We have to solve two Riemann problems

— At $x=0$

As $u_R > u_L$, we expect a rarefaction wave

We seek solutions in the form

$$h = H(\xi) \quad \text{with } \xi = \frac{x}{t}$$

$$H'(-\xi + K H^2) = 0$$

$$\Rightarrow \quad H = \sqrt{\frac{\xi}{K}} = \sqrt{\frac{x}{Kt}}$$

— At $x = l_0$

As $u_R < u_L$, we expect a shock wave

The Rankine Hugoniot equation is

$$\dot{s} = \frac{[\![F(h)]\!]}{[\![h]\!]}$$

with $F(h) = \frac{1}{3} K h^3$

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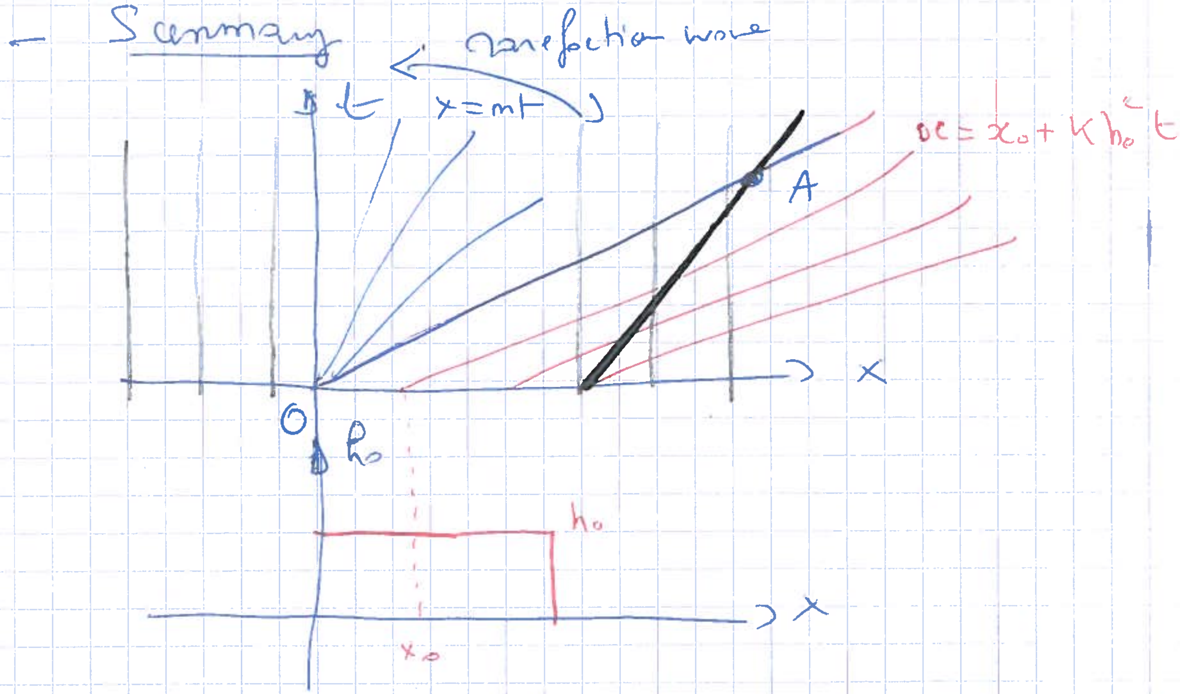
We then find

$$\dot{s} = \frac{F(h_0)}{h_0} = \frac{K}{3} h_0^2$$

The shock propagates at a constant velocity.

- Summary



We observe that the shock wave intersects the fastest characteristic issuing from O. This occurs at time

$$l_0 + \dot{s} t = m_+ t \qquad \text{with } m_+ = K h_0^2$$

$$\Rightarrow \quad t_A = \frac{3}{2} \frac{\mu\, l_0}{\rho g h_0^2 \sin\theta} = \frac{3}{2} \frac{l_0}{K h_0^2}$$

For $t > t_A$, we have to update the shock wave equation

$$\dot{s} = \frac{[\![F(h)]\!]}{[\![h]\!]}$$

$$\dot{s} = \frac{k}{3} h_f^2 \quad \text{but} \quad h_f^2 = \frac{x}{kt} = \frac{s}{kt}$$

$$= \frac{\cancel{k}}{3} \frac{s}{\cancel{k} t}$$

$$\Rightarrow \quad s = x_A \left(\frac{t}{t_A} \right)^{1/3}$$

$$= C \, t^{1/3} \quad \text{with} \quad C = \frac{x_A}{t_A^{1/3}} = \left(\frac{9}{4} k h_0^2 \ell_0^2 \right)^{1/3}$$

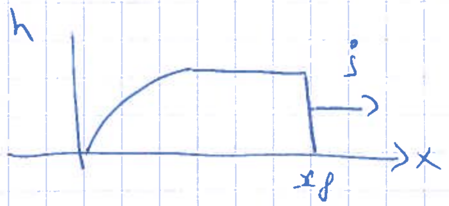
$\boxed{\text{Summary}}$

- For $t < t_A$

$$x_f = \ell_0 + \frac{k h_0^2}{\cancel{3}} t$$

$$h(x,t) = \sqrt{\frac{x}{kt}} \quad \text{for} \quad 0 \leq x \leq \cancel{xn} \, m_+ t$$

$$h(x,t) = h_0 \quad \text{for} \quad m_+ < x \leq x_f$$



- For $t \geq t_A$

$$x_f = \left(\frac{9}{4} k h_0^2 \ell_0^2 \right)^{1/3} t^{1/3}$$

$$h(x,t) = \sqrt{\frac{x}{kt}} \quad \text{for} \quad 0 \leq x < x_f$$