chap6 $\qquad$ 11

exo 7

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial hu}{\cancel{\partial t}} = 0 \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + g\dfrac{\partial h}{\partial x} = 0 \end{cases}$$

BC

$$\begin{cases} Q = uh\big)_0 = \alpha q t^{\alpha-1} \\ F_r = \dfrac{u}{\sqrt{gh}}\Big|_{x=0} = F_0 \\ \dot{x}_f = \beta^2 g h_f \end{cases}$$

Scaling

$$\begin{cases} x \to \lambda x \\ t \to \lambda^a t \\ h \to \lambda^b h \\ u \to \lambda^c u \end{cases}$$

$$\lambda^{b-a}\,\partial_t h + \lambda^{b+c-1}\,\partial_x(hu) = 0$$

$$\Rightarrow \quad a+c=1$$

$$\lambda^{c-a}\,\partial_t u + \lambda^{2c-1}\,u\frac{\partial u}{\partial x} + \lambda^{b-1} g\frac{\partial h}{\partial x} = 0$$

$$c-a = 2c-1 = b-1 \quad \Rightarrow \quad b = 2c$$

$$Q = \lambda^{c+b}\,uh = \alpha q \lambda^{a(\alpha-1)} t^{\alpha}$$

$$\Rightarrow c+b = a(\alpha-1)$$

$$\lambda^{(1-a)2}\,\dot{x}_f = g\lambda^{b} \quad \Rightarrow \quad 2a+b=2$$

$$\begin{cases} a = \dfrac{3}{2+\alpha} \\ b = \dfrac{2(\alpha-1)}{2+\alpha} \\ c = \dfrac{\alpha-1}{\alpha+2} \end{cases}$$

$$\frac{dx}{x} = \frac{dt}{at} = \frac{dh}{bh} = \frac{du}{cu}$$

$$\Rightarrow \quad h = t^{b/a} \, H(\xi)$$

$$\mu = t^{c/a} \, U(\xi) \qquad \text{with } \xi = \frac{x}{t^a}$$

We can forme

$$\begin{cases} U' = \dfrac{F_u}{G_H} \\ H' = \dfrac{F_H}{G_H} \end{cases}$$

see the mathematica notebook

The problem is that the functions depend on ξ and we cannot study the phase portrait $\dfrac{dH}{dU}$.

Here a technique is to set

$$u = \xi \, U(\xi) \, t^{c/a}$$

$$h = \xi^2 H(\xi) \, t^{b/a}$$

By including ξ in the definition of u and h, we can remove ~~the~~ ξ dependence

In doing so, we can form

$$\xi \frac{du}{d\xi} = \dots$$

$$\xi \frac{dH}{d\xi} = \dots$$

and by taking the ratio, we get

$$\frac{dH}{du} = \frac{H\left(6gH - 6u^2 - (2+\alpha) + 3u(3+\alpha)\right)}{-(u-1)u(-2+3u-\alpha) + gh(-2+3u+2\alpha)}$$